Inverse Analysis of Turbulent Weakly Swirling Jets

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Theme

IN the development of turbulence models for complex flow systems, turbulent swirling boundary layers provide a convenient testground. Their equations contain two components of turbulent stress τ_{rz} and $\tau_{r\theta}$, which may be replaced in terms of their associated components of turbulent viscosity μ_{rz} and $\mu_{r\theta}$. These must be specified by a turbulence model prior to the prediction of time-mean velocity and pressure in the flowfield. Recent experimental, inverse, and prediction studies have tried to assess the effect of swirl on these components, and have disputed the isotropy assumption that the $r\theta$-viscosity number $\sigma_{r\theta}=\mu_{rz}/\mu_{r\theta}$, is equal to unity.[1-5] The present work obtains expressions for these components directly from analytical operations being performed on fitted-curve spatial distributions of time-mean values in swirling jet flows, so paralleling an associated numerical inverse technique,[4,5] and bringing new evidence on the swirl flow turbulence problem.

Contents

Analysis: Research experiments have been conducted concerning time-mean measurements on weakly swirling turbulent jet flows.[6-8] The degree of swirl is generally characterized by the nondimensional swirl number S, which is the axial flux of swirl momentum divided by the axial flux of axial momentum times nozzle radius. So that the variation of values with position in the flowfield and degree of swirl is easily calculable, curves have been fitted to the spatial distributions of time-mean values of u, v, w, and p (time-mean axial, radial and swirl velocity in z,r,θ-cylindrical polar coordinates and pressure). It is more convenient and more accurate to use directly the measured flowfield values rather than, for example, deducing some of them from conservation of momentum principles. For the latter technique, results would be particularly sensitive to nozzle exit profile conditions, and it is known[6] that these are not associated with simple plug flow solid body rotation. The v and p fields could be deduced from the equations of conservation of mass and radial momentum. In the case of the former, additional analytical complications arise, and in the case of the latter, it is known that as the degree of swirl increases, the usual boundary-layer form of the equation used to obtain pressure is too approximate. Nor can the full radial momentum equation be used to calculate pressure, for it contains other unknown turbulent stress terms.

How these curvefit parameters vary, based on the sparse amount of experimental data available, with the swirl number S of the jet, is described in the full length paper. The sensitivity of results to uncertainties in these parameters has been considered numerically.[4] It was concluded that the spread/uncertainty ratio for the parameters over the range of swirl numbers considered was such that considerable reliance could be placed on the results. Similar comments are also applicable here.

The axial and swirl momentum equations (for quasi-steady axisymmetric boundary-layer flow neglecting laminar viscosity) may be taken in integral form to isolate the two turbulent stress components, so expressing them directly as functions of the other terms. Substitution of the fitted-curve functions, and carrying out the appropriate analytic differentiation and integration operations (see the Appendices to the full paper) yields simple expressions for the two stress components at any point in the flowfield, in terms of the curve-fit parameters

$$r\tau_{rz} = -r\rho\xi u^2 + r\rho uv + P$$

$$r^2\tau_{r\theta} = -r^2\rho\xi uw + r^2\rho vw \qquad (1)$$

where P is the pressure term considered in detail in the full paper, and $\xi = r/(z+a)$ where a is apparent origin distance upstream from nozzle. Invoking stress-strain constitutive assumptions gives expressions for the two associated components of turbulent viscosity. Then the $r\theta$-viscosity number $\sigma_{r\theta}$, mixing lengths L and l, and mixing length parameter λ may be calculated from

$$\sigma_{r\theta} = \mu_{rz}/\mu_{r\theta}$$

$$\mu_{rz} = \rho L r_{0.01} u_m$$

$$\mu_{rz} = \rho l^2 \{ (\partial u/\partial r)^2 + (r\partial(w/r)/\partial r)^2 \}^{1/2}$$

$$l = \lambda r_{0.01} \qquad (2)$$

where u_m is the station maximum axial velocity and the suffix 0.01 refers to the position where $u/u_m = 0.01$.

Results and Discussion: The results presented show that the technique has strong potential for boundary-layer flows in which there is difficulty obtaining accurate turbulence measurements in comparison to the relative ease of measuring time-mean values. The technique is applied here, however, only to inert swirling jet flow data, as described earlier. The two shear stress components have been computed for various values of S, ξ, and z/d where d is nozzle diameter. Generally both jet-normalized components (obtained by division by ρu_{mo}^2) decrease with z/d, but the τ_{rz} values decrease, whereas the $\tau_{r\theta}$ values increase, with S at z/d values greater than 6. Absolute τ_{rz} values are an order of magnitude larger than $\tau_{r\theta}$ values. On the other hand station-normalized values (division by ρu_m^2) generally increase with swirl, and though the $r\theta$-values generally decrease with axial distance, the rz-values remain fairly constant.

The decrease with swirl of the absolute τ_{rz} values, and turbulent intensity in the fully developed region (approximately $z/d>6$) is a somewhat surprising result which explains a phenomenon which has been noted in flames with swirl: if fuel was not fully burned in the initial region, considerable difficulty was found in burning the fuel residue in the fully developed region of the flame. Moreover, extremely close to the jet nozzle, the trend of the τ_{rz} and turbulence intensity

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Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Boundary Layers and Convective Heat Transfer-Turbulent.

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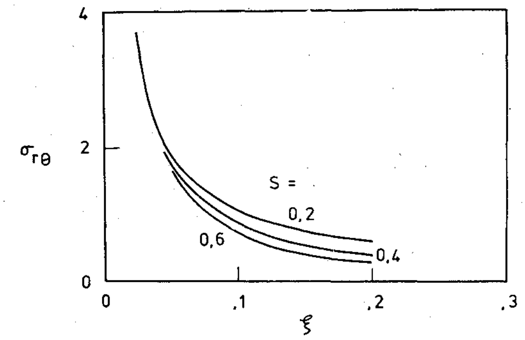

Fig. 1 Lateral distribution of $r\theta$-viscosity number $\sigma_{r\theta} = \mu_{rz}/\mu_{r\theta}$ at $z/d = 6$.

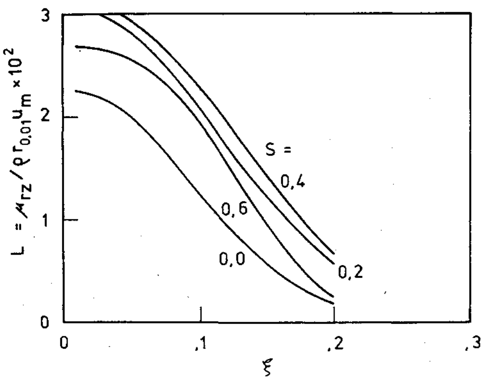

Fig.2 Lateral distribution of nondimensionalized viscosity $L = \mu_{rz}/(\rho r_{0.01} u_m)$ at $z/d = 6$.

values is the reverse, that is to increase with swirl. Thus in order to utilize the increased rate of mixing in a swirling jet, mixing must take place in the region close to the nozzle exit. The shear stress lateral distributions are also found to exhibit their maximum values at locations corresponding to maximum radial gradients of axial velocity. Hence, for good combustion system mixing, for example, fuel should be injected into these regions of maximum velocity gradients and maximum shear stress.

By invoking the stress-strain relations, the two turbulent viscosity components μ_{rz} and $\mu_{r\theta}$ may be obtained independently from the two shear stress components. It is possible to look at the $r\theta$-viscosity number $\sigma_{r\theta}$ at various locations in the flowfield for various degrees of swirl. The lateral profile is shown in Fig. 1 for the axial station $z/d = 6$, though there is not much downstream variation. Nor is there seen to be much variation with swirl. The figure shows the remarkable degree of nonisotropy which occurs in some regions of swirling flows. At a given axial station, $\sigma_{r\theta}$ is generally larger than unity in the central region $\xi < 0.1$, and less than unity in the outer mixing region, indicating that $\mu_{r\theta}$ is considerably less than μ_{rz} in the central part of the mixing region thus confirming the trend found elsewhere.[1-5]

The interesting part of the flowfield is wider and shorter in a swirling jet as compared with a nonswirling jet, and this is bound up with the value of turbulent viscosity μ_{rz}, whose nondimensional value is L in Eq. (2). In the full paper other characterizations are considered, but this L varies least with swirl and position, making it in some sense most useful. Lateral distributions generally show maxima in the central region, and reduction to laminar values in the outer regions. Figure 2 illustrates at $z/d = 6$, the effect of swirl on L, (values generally increasing with swirl, and changing a little in the downstream direction, though after the initial region where pressure gradients are significant, this change is fairly small). Values are found to be very similar to those found previously,[5] and show the progressive increase in values

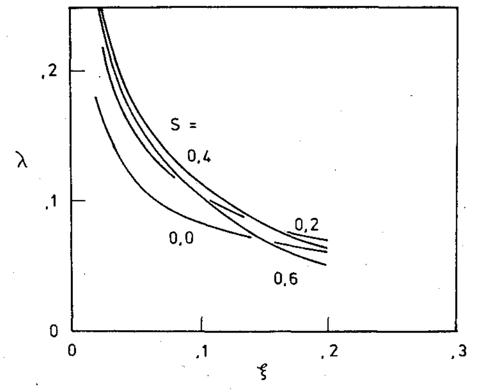

Fig. 3 Lateral distribution of mixing length parameter λ at $z/d = 6$.

above those found in nonswirling jets. Values decrease toward the edge of the jet as ξ increases, as compared with the constant cross-section value (of approximately 0.00196 ÷ tan 12° = 0.0092) obtained for a nonswirling jet,[9] through the use of non-Gaussian axial velocity radial distribution. This trend is expected – the constant viscosity assumption is associated with an axial velocity profile which is too peaked near the axis. Use of the more rounded (i.e., less peaked) Gaussian distribution is associated with higher viscosity values in the central region, and lower values in the outer region as the figure reveals. This is entirely consistent with previous numerical inverse studies.[4,5] Two important phenomena have emerged: the effect of swirl enhances the station-normalized primary components of stress and viscosity (τ_{rz} and μ_{rz}), and the distributions of stress and viscosity are nonisotropic.

Relating the primary component of turbulent viscosity to the second invariant of the time-mean flow rate of strain tensor, generates the mixing length l and parameter λ by way of Eq. (2). How λ varies with swirl and lateral position is revealed in Fig. 3 at $z/d = 6$, and similar results are found at other axial stations. For nonswirling jets it has been found that good predictions can be made[2] with the assumption of a constant mixing length parameter equal to about 0.08. An examination of the figure shows that there is generally an increase of λ with swirl and proximity to the center. On the basis of these results it may be concluded that good predictions can be made for weakly swirling jet flows downstream of the initial region with the simple use of a larger mixing length parameter, progressively increased with degree of swirl.

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